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# Bayesian Method for Linear Regression Modeling; Simulation Study with Normality Assumption

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## Abstrak

Penelitian ini bertujuan untuk melakukan pemodelan regresi dengan estimasi parameter menggunakan metode Bayes. Metode Bayes merupakan analisis penduga parameter dengan mencari solusi dari distribusi posterior. Data yang digunakan sebanyak 100 data dari hasil simulasi. Prior yang digunakan adalah prior normal. Variabel yang digunakan terdiri dari satu variabel dependent (Y) dan tiga variabel independent ($X_1$, $X_2$, dan $X_3$) yang berdistribusi normal. Diperoleh model yang menghasilkan parameter yang signifikan di semua variabel bebas berdasarkan selang kepercayaan. Interpretasi model yaitu jika terjadi kenaikan satu satuan pada $X_1$, maka akan terjadi kenaikan sebesar 1.950 pada Y. Jika terjadi kenaikan satu satuan pada $X_2$, maka Y akan mengalami penurunan sebesar 2.950. Jika terjadi kenaikan satu satuan pada ($X_3$), maka akan terjadi kenaikan sebesar 0.940 pada Y. Kekonvergenan parameter dilihat dari *density plot* dan *trace plot* yang telah mengikuti distribusi normal dengan melihat parameter sudah berada pada selang kepercayaan. Indikator kebaikan model yang digunakan adalah MSE. Nilai MSE yang diperoleh nilai sebesar 27.17. Nilai MSE tersebut tergolong kecil jika dilihat dari variansi dan skala data. Nilai ini membuktikan model ini sudah bagus untuk memodelkan kasus ini.

**Kata Kunci: regresi linear, Metode Bayes,MSE, prior, posterior.**

## Abstract

This study aims to conduct regression modeling with parameter estimation using the Bayes method. The Bayes method is an analysis of parameter estimation by looking for solutions from the posterior distribution. The data used are 100 data from simulation results. The prior used is normal prior. The variables used consist of one dependent variable (Y) and three independent variables ($X_1$, $X_2$, dan $X_3$) which are normally distributed. A model is obtained that produces significant parameters in all independent variables based on the confidence interval. The interpretation of the model is that if there is a one unit increase in $X_1$, there will be an increase of 1,950 in Y. If there is a one unit increase in $X_2$, then Y will decrease by 2,950. If there is a one-unit increase in ($X_3$), there will be an increase of 0.940 in Y. The convergence of parameters is seen from the density plot and trace plot which has followed the normal distribution by looking at the parameters already in the confidence interval. The indicator of model goodness used is MSE. The MSE value obtained is 27.17. The MSE value is relatively small when viewed from the variance and scale of the data. This value proves that this model is good for modeling this case.

**Keywords:** *Bayesian Method, MSE, linear regression, prior, posterior*

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# INTRODUCTION

Regression analysis is one of the statistical techniques used to predict the relationship and influence between a dependent variable and one or more independent variables[1]. Dependent variables are variables that are influenced or dependent on other variables in a study or experiment. This variable is the result or effect of changes that occur in the independent variable. An independent variable is a variable that affects or causes changes in other variables in a study or experiment. This variable can be manipulated or controlled by the researcher to see its impact on the dependent variable.

There are various types of regression, including simple linear regression, multiple linear regression, logistic regression, and non-linear regression. Simple linear regression is used when there is one independent variable affecting the dependent variable, while multiple linear regression is used when there is more than one independent variable contributing to the dependent variable. This simple regression method is also carried out on modeling various cases such as those carried out by researchers [2],[3],[4]. In many cases, the available data cannot explain the relationship exactly, or the model used may have significant uncertainty.

The main purpose of regression analysis is to understand the extent to which the independent variable can explain the dependent variable, measure the strength of the relationship between variables, and make predictions based on the model that has been formed. If the data will be modeled using linear regression, there are several assumptions that must be met so that the model provides valid results and can be interpreted properly. The following are these assumptions linearity (The relationship between the independent variable (predictor) and the dependent variable (response) must be linear, does not contain autocorrelation (the residuals (errors) of the regression model are random and do not show a systematic pattern, Homoscedasticity (the variance of the residuals should be constant for all values of the independent variable), Normality of Residuals (Residuals should be normally distributed for statistical inference can be done correctly), No Multicollinearity (independent variables should not have a very high correlation with each other). However, in many real-world applications, uncertainty in data and models is often difficult to ignore. Therefore, Bayesian-based approaches offer a way to handle this uncertainty more explicitly.

Bayes method is method to combine prior knowledge with likelihood function or evidence to produce more accurate decisions. The method is based on Bayes Theorem, which describes how to calculate inverse probabilities, i.e. the probability of a hypothesis being true given observed data [7]. The distribution is called the prior distribution (initial distribution of the data) with the probability density function of the data we observe [9]. The posterior distribution is the product of the likelihood function or joint probability density function multiplied by the prior distribution [10].

The average of this posterior distribution will be the parameters of the model that will be generated.

The advantage of the Bayes method over the simple parameter estimation method (Ordinary Least Square) is that the parameter is considered a random variable that has a distribution [8]. Some studies using regression analysis with parameter estimation using the Bayesian method include [11],[12],[13]. Based on the reading of articles related to Bayesian modeling, the author is interested in conducting regression modeling with the Bayesian method with simulated data from the normal distribution.

# METHODOLOGY

## Linear Regression

If a vector $Y = (y_1, y_2, \dots, y_n)'$ is declared as the dependent variable and $X = (x_1, x_2, \dots, x_n)'$ is defined as the independent variable, then the hypothesis model for linear regression with $n$ samples and $k$ predictors for $i = 1,2, \dots, n$ is written as [2]:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \varepsilon_i \quad (1)$$

Where:

$Y$ is dependent variable with size *n x 1.*

$X_1, X_2, \dots, X_n$ is the independent variable that affects Y with size *n* x *(k+1)*

$\beta_0$ is the intercept or constant (the value of $Y$ when all X = 0), and

$\beta_1$, $\beta_2$,..., $\beta_n$ is a regression coefficient that measures the effect of each independent variable on the dependent variable. Regression coefficient with size *(k+1) x 1*

$\varepsilon_i$ is error with size *n x 1.*

## Bayesian Methods

Bayes theorem is the foundation of the Bayesian method, developed by Thomas Bayes in the 18th century. This theorem describes how we can update the probability of a hypothesis or parameter based on newly obtained evidence. Bayes theorem is formulated by equation below [7]:

$$P((A|B) = \frac{P((B|A)P(A)}{P(B)} \quad (2)$$

Where:

$P((A|B)$ is posterior distribution

$P((B|A)$ is likelihood function

$P(A)$ is prior distribution

$P(B)$ is probabilitas marginal (total probability from data (B) not depent to A.

The Bayesian concept above will be used as the basis for estimating parameters in the Bayesian concept. The Bayes method combines the likelihood function with the prior distribution of the parameters to obtain the posterior distribution which is the basis for estimating the parameters.

**Definition 1.** [14] Joint probability density function of random variables $X_1, X_2, \dots, X_n$ calculated at $x_1, x_2, \dots, x_n$ is $f(x_1, x_2, \dots, x_n|\beta)$ which is the likelihood function. For a given value of $x_1, x_2, \dots, x_n$ the likelihood function is a function of the parameter $\beta$ which can be denoted by $L(\beta)$. If $X_1, X_2, \dots, X_n$ are mutually independent random samples of $f(x;\beta)$ then:

$$L(\beta) = f(x_1;\beta)\, f(x_2;\beta)\, \dots f(x_n;\, \beta) \quad (3)$$

$$= \prod_{i=1}^{n} f(x_i, \beta)$$

The probability density function of normal distribution is as follows [14]:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (4)$$

Where:

$\mu$ is mean of data (determine the posisition of the center of the distribution).

$\sigma^2$ is variance (determines how far the data is spread from the mean)

The prior distribution is the initial distribution that provides information about the parameters to be estimated. In estimating a parameter value, the prior distribution can be chosen subjectively by the researcher. The prior distribution is divided into two, namely [10],[15]:

a. Related to the distribution form of the results of identifying the data pattern obtained from the likelihood function, namely:
   1. Conjugate prior distribution. This prior distribution is determined based on the choice of priors in a model considering the likelihood function.
   2. Non-conjugate prior distribution. Giving priors to the model ignores the pattern of forming the likelihood function.

b. Related to previous information related to the determination of each parameter in the prior distribution pattern, namely:
   1. Informative prior distribution. This prior distribution refers to the assignment of parameters from the prior distribution that has been selected, both the conjugate prior distribution and the non-conjugate prior.
   2. Non-informative prior distribution. This prior distribution is not based on existing data or a prior distribution that contains no information about the parameters.

In this study, the priors used are conjugate priors, namely normal priors and Exponential Priors. The first; The normal distribution, also known as the Gaussian distribution, is a probability distribution that has a symmetrical bell shape. It is very commonly used in many fields, including Bayesian statistics, as a prior distribution. In the context of Bayesian statistics, the normal prior distribution is used to represent initial beliefs about the values of the parameters to be estimated, before the data is obtained. In the context of priors, the normal distribution is often used due to its simplicity and the large number of natural phenomena that approximate the normal distribution. The normal prior distribution allows for a smooth spread of beliefs and can be easily combined with the likelihood to produce a posterior distribution that also has a normal shape. The prior Normal is written by $\boldsymbol{N} \sim (\boldsymbol{b_0}, \boldsymbol{B_0})$ [16]**.**

After we determine the likelihood function and the prior distribution, we will then estimate the mean of the posterior distribution. The posterior distribution is the distribution of the parameter The posterior distribution is the distribution of the parameter $\boldsymbol{\beta}$ that combines the information from the likelihood function with the prior information about the parameter expressed by the prior distribution through Bayes' theorem. that combines the information from the likelihood function with the prior information about the parameter expressed by the prior distribution through Bayes theorem.

**Definition 2.** The conditional probability density function of the parameter $\beta$ given observations $x = x_1, x_2, \dots, x_n$, is expressed as the posterior probability density function given by [7]:

$$f(\beta|x) = \frac{L(\beta)f(\beta)}{\int_{-\infty}^{\infty} L(\beta)f(\beta)d\beta} \tag{5}$$

Since the function in the denominator tends to be constant because it does not depend on the value of β, the posterior probability density function can be written as follows [7]:

$$f(\beta|x) \propto L(\beta)f(\beta) \tag{6}$$

**Definition 3:** [17] The mean of the posterior distribution $f(\beta|x)$ expressed by $\hat{\beta}$ is referred to as the Bayes estimator for $\beta$.

## Data

The data used in this research is simulated data. This research data consists of three independent variable ($X_1, X_2, and\ X_3$). This generated as many as 100, from a normal distribution with $X_1$ (mean = 0, standard deviation = 1) or can be written $N(\mu, \sigma^2) = N(0,1)$, $X_2$ (mean = 0, standard deviation = 2) or can be written $N(\mu, \sigma^2) = N(0,4)$, $X_3$ (mean = 0, standard deviation = 4) or can be written $N(\mu, \sigma^2) = N(0,16)$. So dependent variable is written as:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon_i \tag{7}$$

Model data used in the simulation data by setting

$$Y = 5 + 2X_1 - 3X_2 + X_3 + \varepsilon_i \tag{8}$$

Hereinafter referred to as the research variable comes from the Gaussian family. The prior used in this study is the Normal prior. In the Bayesian method we use Monte Carlo Markov Chain (MCMC) to estimate the parameters. The number of Markov chains used is four Markov chains, the number of iterations is 2000 iterations and the repetition process is 1000 times, and the credible interval (CI) used is 95%. This research uses R software.

# Indicator of Model Goodness

## Mean Square Error (MSE)

MSE is determined by calculating the average of the squares of the resulting regression model. The MSE value can be determined in the following equation[7]:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \qquad (9)$$

Where $y_i$ is the-$i$th observation value and $\hat{y}_i$ is the estimated value of the i-th estimation result.

# RESULT AND DISCUSSION

## Estimated Parameter with Bayesian Methods

The results of the data analysis using the Bayesian method to estimate the research parameters can be seen in the table below. The parameter to be estimated is $\beta_0$, $\beta_1$, $\beta_2$ and $\beta_3$ and scale parameter $\sigma$. Because there is only one independent variable in this study, namely the price variable.

**Table 1. Estimated Parameter $\beta_0$, $\beta_1$, $\beta_2$ and $\beta_3$ with Bayesian Methods**

| Parameter | Esimated value of $\beta$ | Estimated Error | Credible Interval 95% | |
|---|---|---|---|---|
| | | | Lower 95% | Upper 95% |
| $\beta_0$ (Intercept) | 4.980* | 0.110 | 4.770 | 5.200 |
| $\beta_1$(Coefficient ($X_1$) | 1.950* | 0.120 | 1.720 | 2.180 |
| $\beta_2$(Coefficient ($X_2$) | -2.950* | 0.110 | -3.170 | -2.740 |
| $\beta_3$(Coefficient ($X_3$) | 0.940* | 0.110 | 0.720 | 1.700 |
| $\sigma$ (Scale Paramater) | 1.060 | 0.080 | 0.930 | 1.230 |
| * signifikancy for $\alpha = 0.05$ | | | | |

So the regression model with parameter estimation using the Bayesian method is as follows:

$$Y = 4.980 + 1.950X_1 - 2.950X_2 + 0.940X_3 \qquad (10)$$

Based on the model presented in equation (9), it can be seen that if there is a one unit increase in $X_1$, there will be an increase of 1,950 to Y if other independent variables are constant. If there is a one unit increase in ($X_2$), Y will decrease by 2,950 if other independent variables are constant. If there is a one unit increase in ($X_3$), there will also be an increase in Y by 0.940 if other independent variables are constant. All independent variables have a significant influence on Y, it can be seen from the credible interval (CI) in Table 1.

The convergence of parameters in this Bayesian method can be seen from the trace plot and density plot. The image of the convergence of each parameter can be seen in the figure below:

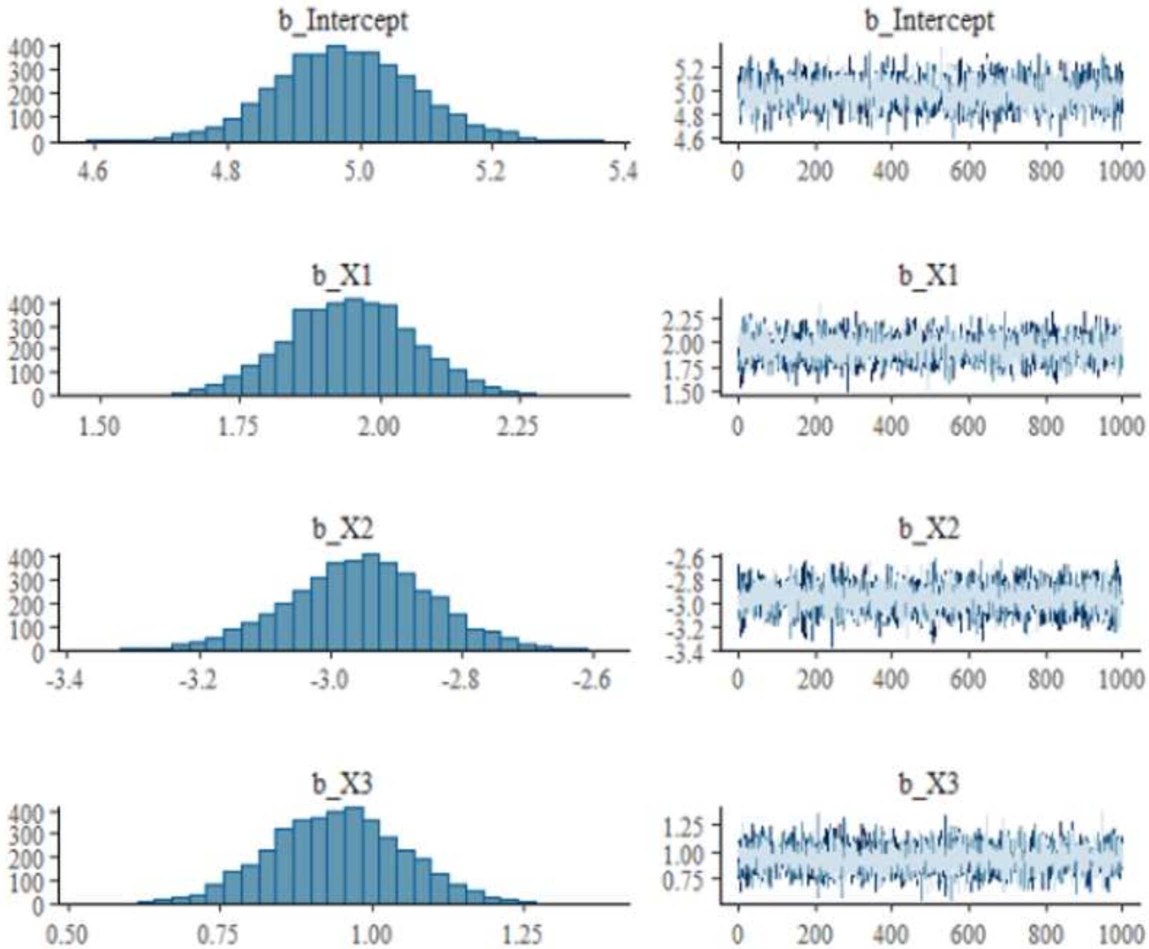


**Figure 1. Trace Plot and Density Plot of Parameters**

**Figure 1** shows that each average of the parameters estimated from the posterior distribution solution has converged at the 1000*th* iteration. Each parameter has converged around the mean. In the Bayes method, the scale parameter refers to a parameter in a probability distribution that controls how wide or narrow the distribution is. This parameter is often used in the context of distribution priors and likelihoods in Bayesian updating. The convergence of the scale parameters can be seen in Figure 2 below:

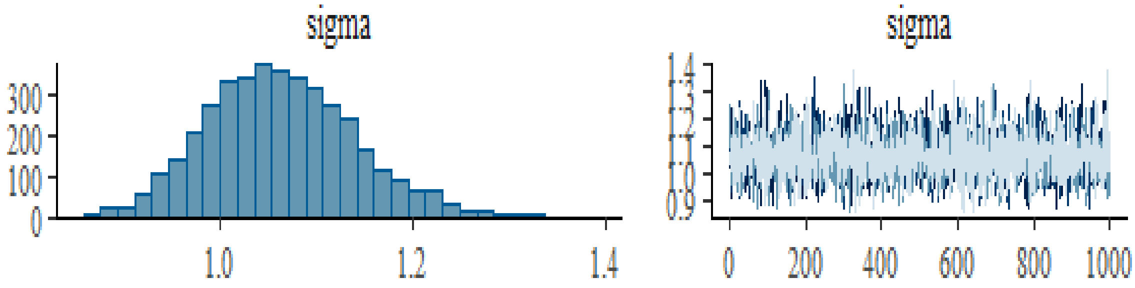


**Figure 2. Trace Plot and Density Plot of Parameters scale**

Based on **Figure 2**, it can be seen that the scale parameter has also converged and formed a normal curve. With this convergence, the spread of parameters is already around the mean and stable, so there is no pattern of very large and wide spread. This means that the error will be smaller than the results obtained.

The MSE value is used as an evaluation of the parameter estimation method. Formula MSE is writte below:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

**Table 2. The error calculation to get the MSE value**

| No | Y | $X_1$ | $X_2$ | $X_3$ | $\hat{Y}$ | Error ($\varepsilon$) |
|---|---|---|---|---|---|---|
| 1 | 7.493837 | -1.95777 | 2.259671 | -1.75215 | -7.150698997 | 14.64454 |
| 2 | 5.473732 | 0.031658 | -1.40299 | -0.02643 | 9.15571781 | -3.68199 |
| 3 | 10.16094 | 0.04359 | -0.28095 | -1.21709 | 4.749730666 | 5.411206 |
| 4 | 9.399987 | -0.57357 | -0.4026 | -1.04997 | 4.062228574 | 5.337759 |
| 5 | -2.66823 | 0.110317 | 0.46903 | -0.17535 | 3.64664661 | -6.31488 |
| 6 | 4.492128 | 0.597491 | 3.055358 | 0.694548 | -2.215325297 | 6.707454 |
| 7 | 8.942793 | 1.180964 | -0.92943 | -1.66048 | 8.463834387 | 0.478958 |
| 8 | 10.35276 | 0.111378 | -0.30092 | 0.149441 | 6.225372613 | 4.127383 |
| 9 | -1.37918 | -1.1023 | -0.27048 | -1.35376 | 2.355888642 | -3.73506 |
| 10 | 5.612117 | -0.93081 | 0.246466 | -0.78559 | 1.699392311 | 3.912725 |
| 11 | 1.874728 | 1.482939 | -1.02625 | -1.22136 | 9.751092616 | -7.87636 |
| 12 | 4.231727 | 1.68623 | -0.67629 | 0.384691 | 10.6247983 | -6.39307 |
| 13 | 7.334001 | 1.593305 | 0.96291 | 0.29201 | 5.520848741 | 1.813152 |
| 14 | 4.466896 | 0.300022 | -0.01824 | 0.9149 | 6.478871217 | -2.01198 |
| 15 | 5.862143 | -0.46423 | -0.42646 | -1.17212 | 4.231007199 | 1.631136 |
| 16 | 6.86501 | 0.462543 | -0.43293 | 0.421947 | 7.555733633 | -0.69072 |
| 17 | 8.665246 | -1.2134 | -1.1494 | -0.3239 | 5.700140478 | 2.965106 |
| 18 | 4.328718 | 0.721599 | 0.924145 | -0.32532 | 3.35508815 | 0.97363 |
| 19 | 7.091067 | -0.74506 | -1.35425 | 0.607696 | 8.093406704 | -1.00234 |
| 20 | 4.270481 | -0.32308 | 0.584675 | -1.17843 | 1.517484846 | 2.752996 |
| 21 | 6.753479 | 0.059817 | -0.3385 | 0.758759 | 6.808459691 | -0.05498 |
| 22 | 4.325959 | 1.723903 | -1.33517 | 1.383083 | 13.5804685 | -9.25451 |
| 23 | 1.776591 | -0.79863 | 0.15139 | 0.354785 | 3.309571791 | -1.53298 |
| 24 | 2.874535 | 0.851338 | 0.559629 | 0.507732 | 5.466472949 | -2.59194 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 12.80063 | 0.542575 | -0.18759 | 0.0454 | 6.634103257 | 6.166525 |
| 26 | 8.999907 | -0.86671 | 0.686112 | -0.68709 | 0.620026391 | 8.379881 |
| 27 | 9.119465 | -0.93388 | 1.502128 | -1.65139 | -2.824640054 | 11.94411 |
| 28 | 7.304207 | -0.22341 | 0.557016 | -0.42017 | 2.5061891 | 4.798018 |
| 29 | 3.855414 | 0.719875 | 0.134672 | -1.56482 | 4.515542219 | -0.66013 |
| 30 | -3.37365 | 0.100174 | -0.17779 | 1.121458 | 6.753983475 | -10.1276 |
| 31 | 4.331707 | 0.097359 | -1.18731 | -0.31911 | 8.372455591 | -4.04075 |
| 32 | 7.7677 | 0.483282 | 1.315099 | 0.074742 | 2.11311546 | 5.654585 |
| 33 | 5.967386 | 0.361253 | 1.616976 | -1.13835 | -0.155683106 | 6.123069 |
| 34 | 11.31386 | -0.43165 | -0.96391 | 0.623169 | 7.567592205 | 3.746272 |
| 35 | 7.653809 | -1.3399 | -2.43132 | -0.4193 | 9.145422002 | -1.49161 |
| 36 | 7.654556 | -0.42144 | 0.673141 | 0.390279 | 2.539296133 | 5.11526 |
| 37 | 0.985543 | -0.41471 | -0.18772 | 0.158321 | 4.873896343 | -3.88835 |
| 38 | 2.17487 | 1.037939 | 0.494431 | -1.45898 | 4.173967218 | -1.9991 |
| 39 | 5.203996 | 0.759099 | -0.43304 | 1.203798 | 8.869266652 | -3.66527 |
| 40 | 6.159487 | -2.17599 | -0.01141 | -1.37198 | -0.519176328 | 6.678663 |
| 41 | -0.14915 | -0.14869 | 1.42672 | -0.65389 | -0.133425809 | -0.01573 |
| 42 | 11.9895 | 0.485987 | 0.046091 | -0.08024 | 5.716277718 | 6.273221 |
| 43 | 3.782761 | -1.61153 | -0.78519 | 0.435555 | 4.563243912 | -0.78048 |
| 44 | 1.705755 | -0.25327 | -0.62013 | -0.7721 | 5.589728037 | -3.88397 |
| 45 | 8.434972 | 0.812346 | -0.25432 | 0.17631 | 7.480060497 | 0.954911 |
| 46 | 5.762092 | 1.498196 | 0.49474 | 1.918545 | 8.245433001 | -2.48334 |
| 47 | 7.207767 | -0.36079 | -0.25068 | 1.309963 | 6.247333754 | 0.960433 |
| 48 | 6.568583 | 0.947742 | -0.1441 | 1.387866 | 8.557794325 | -1.98921 |
| 49 | 4.541173 | 1.857526 | 0.629972 | -2.62325 | 4.277905489 | 0.263268 |
| 50 | 7.223227 | -0.83153 | 1.101952 | 0.490105 | 0.568459679 | 6.654767 |
| 51 | 5.122398 | 0.926354 | -0.73044 | 1.537517 | 10.38646253 | -5.26406 |
| 52 | 5.674325 | -0.41128 | 1.014928 | -1.57889 | -0.300191733 | 5.974517 |
| 53 | 3.335232 | -0.11141 | 0.69977 | -1.07893 | 1.684245789 | 1.650986 |
| 54 | 10.04959 | -0.31074 | -1.25236 | 0.286203 | 8.337565551 | 1.712021 |
| 55 | 9.242568 | -1.24601 | -0.11131 | 0.341314 | 3.199482674 | 6.043086 |
| 56 | 4.988606 | 0.278403 | -0.41701 | -0.65945 | 6.133169015 | -1.14456 |
| 57 | 8.804447 | 0.956421 | 0.115797 | 1.047527 | 7.488094695 | 1.316353 |
| 58 | 7.687318 | 1.859939 | 0.716866 | -0.51567 | 6.007394794 | 1.679923 |
| 59 | 4.495627 | 0.260522 | -0.47799 | -1.58709 | 5.406214251 | -0.91059 |
| 60 | 0.081155 | 0.152424 | 0.747715 | 0.077609 | 3.14441912 | -3.06326 |
| 61 | 7.884262 | 0.560912 | -1.27851 | -0.0541 | 9.794538961 | -1.91028 |
| 62 | -4.4354 | -0.50202 | -0.30704 | 0.043896 | 4.948075594 | -9.38347 |
| 63 | 7.621445 | -1.23221 | 0.357044 | -0.37084 | 1.175315032 | 6.44613 |
| 64 | 9.303093 | -0.24446 | 0.2342 | -0.087 | 3.73062042 | 5.572473 |
| 65 | 2.907732 | 0.351709 | -0.36067 | 1.401159 | 8.046906504 | -5.13917 |
| 66 | 6.121409 | 0.028637 | 2.026345 | -1.59345 | -2.439716092 | 8.561125 |
| 67 | 5.023332 | -0.21894 | -0.31331 | 0.72376 | 6.157672777 | -1.13434 |
| 68 | 7.398971 | 0.801586 | 0.032282 | 0.481841 | 6.900790857 | 0.49818 |
| 69 | 7.261932 | -1.05903 | 1.545071 | -0.00417 | -1.646996118 | 8.908928 |
| 70 | 10.59463 | 1.204692 | 0.016752 | 1.603737 | 8.787243617 | 1.807382 |
| 71 | 5.131838 | 1.796256 | 0.487433 | -0.66769 | 6.417147846 | -1.28531 |
| 72 | 4.190737 | -0.96131 | 0.345241 | -0.99018 | 1.156217372 | 3.03452 |
| 73 | 7.829464 | -0.59591 | 1.16065 | -2.06693 | -1.548868484 | 9.378333 |
| 74 | 12.6059 | 0.546073 | -0.05147 | 0.255634 | 6.436969903 | 6.168933 |
| 75 | 1.340799 | -0.70883 | 0.623585 | -0.46341 | 1.322587987 | 0.018211 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 76 | 8.57785 | -0.5797 | 1.285937 | 1.157399 | 1.144024282 | 7.433826 |
| 77 | 3.146455 | 1.100292 | -1.86528 | -1.57792 | 11.14491655 | -7.99846 |
| 78 | -0.26275 | 1.584933 | 0.737311 | -0.38731 | 5.531482953 | -5.79423 |
| 79 | 7.217729 | 2.007266 | 0.737783 | -2.23846 | 4.613556337 | 2.604172 |
| 80 | 7.925866 | -1.69603 | -1.9126 | 1.056471 | 8.30797827 | -0.38211 |
| 81 | 3.495111 | -0.08679 | 1.520751 | 0.545016 | 0.83684997 | 2.658261 |
| 82 | 5.868384 | 0.055419 | -0.25785 | -0.45211 | 5.423735214 | 0.444649 |
| 83 | 1.43128 | 0.909774 | 0.413294 | 0.619974 | 6.117617477 | -4.68634 |
| 84 | 13.08263 | -0.92538 | -0.40753 | -1.27318 | 3.180931782 | 9.9017 |
| 85 | 8.420145 | 1.854163 | -0.0021 | 1.136805 | 9.670416391 | -1.25027 |
| 86 | 4.896197 | -1.56144 | -1.46989 | -0.29201 | 5.996889264 | -1.10069 |
| 87 | 1.007169 | -1.84452 | 2.461391 | 0.127462 | -5.758108253 | 6.765277 |
| 88 | 5.678903 | 1.290958 | 3.521557 | -0.45817 | -3.321899297 | 9.000802 |
| 89 | 6.713252 | -0.91507 | 0.163398 | -0.52926 | 2.216075031 | 4.497177 |
| 90 | -0.38597 | -0.39347 | -0.22171 | 0.266671 | 5.11744775 | -5.50342 |
| 91 | 5.827565 | -1.18572 | -0.16019 | 1.071041 | 4.14719541 | 1.680369 |
| 92 | 0.071248 | 1.595401 | 0.726019 | -0.35259 | 5.617840482 | -5.54659 |
| 93 | 10.96947 | 2.549962 | 0.429167 | -0.5684 | 8.15209156 | 2.817377 |
| 94 | 7.594182 | 1.011264 | -0.57812 | -0.22719 | 8.443872234 | -0.84969 |
| 95 | 6.050162 | -1.58716 | -0.61084 | 0.392025 | 4.055523346 | 1.994639 |
| 96 | -0.50274 | -1.15539 | -1.35269 | 1.87037 | 8.475564323 | -8.9783 |
| 97 | 1.846149 | -1.48637 | -0.90189 | 1.813156 | 6.446528409 | -4.60038 |
| 98 | 7.564623 | 0.341272 | 0.742119 | -0.90349 | 2.606946085 | 4.957677 |
| 99 | 9.30966 | -0.43311 | 1.332988 | 0.046964 | 0.247272088 | 9.062388 |
| 100 | -0.62156 | -0.51894 | -1.02884 | -0.46328 | 6.567652437 | -7.18921 |

Based on table 2 above, the calculation for the MSE value is as follows:

$$MSE = \frac{1}{100}\sum_{i=1}^{100}(y_i - \hat{y}_i)^2$$

$$MSE = \frac{1}{100}(2717.452)$$

$$MSE = 27.17$$

Based on the above calculations, the MSE value of 27.17 is relatively small when compared to the variance of the data and scale data.

# CONCLUSION

Based on the data analysis above, it can be concluded that this study produces parameters that are quite convergent around the mean resulting from the estimation of the posterior distribution. The resulting variables have a significant relationship and influence when viewed from the Credible Interval. The indicator of model goodness using MSE value of 27.71. This value is small enough to represent an indicator of model goodness. The model produced in the study was generated from simulated normally distributed data, with normal priors as well. From the MSE results, the Bayesian method can be proposed as a parameter estimation for the case of data in the field, with various prior assumptions chosen.

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